Primer on Inventory Management – Solutions

Demand Estimation

Solution to Exercise 1:

i. See spreadsheet. $\hat{\mu} = 19.87$, $\hat{\sigma} = 5.01$. Demand can be reasonably modeled by a Normal distribution.

ii. Optimal S:

$$S^\star = F^{-1}\left(\frac{5}{5+0.2}\right) = F^{-1}(0.9615) = 19.87 \cdot (LT+1) + 1.769 \cdot \sqrt{LT+1} \cdot 5.01.$$

For $LT = 50$:

$$S^\star = 19.87 \cdot (51) + 1.769 \cdot \sqrt{51} \cdot 5.01 \approx 1077$$

For $LT = 100$:

$$S^\star = 19.87 \cdot (101) + 1.769 \cdot \sqrt{101} \cdot 5.01 \approx 2097$$

For $LT = 20$:

$$S^\star = 19.87 \cdot (21) + 1.769 \cdot \sqrt{21} \cdot 5.01 \approx 458$$

iii. Optimal Cost:

$$\begin{aligned} Z(S^\star) &= (h+p) \cdot f_{N(0,1)}(z) \cdot \sigma_{LT+1} \\ &= 5.2 \cdot 0.0835 \cdot 5.01 \cdot \sqrt{LT+1} \end{aligned}$$

For $LT = 50$:

$$Z(S^\star) = 15.54$$

For $LT = 100$:

$$Z(S^\star) = 21.87$$

For $LT = 20$:

$$Z(S^\star) = 9.97$$

iv. Optimal S:

$$S^\star(LT) = 19.87 \cdot (LT+1) + 1.769 \cdot \sqrt{LT+1} \cdot 5.01.$$

Optimal $E\left[S - Y^{LT+1}\right]^+$:

We know that

$$E\left[S - Y^{LT+1}\right]^+ = S - \mu_{LT+1} + E\left[Y^{LT+1} - S\right]^+,$$

therefore, we first compute expected backorders:

$$\begin{aligned}
E\left[Y^{LT+1} - S\right]^+ &= \sum_{y=S}^{\infty} (y - S) \cdot f_{LT+1}(y)\, dy \\
&= L\left(\frac{S^\star - \mu_{LT+1}}{\sigma_{LT+1}}\right) \cdot \sigma_{LT+1} \\
&= L\left(\frac{\mu_{LT+1} + F^{-1}(CR) \cdot \sigma_{LT+1} - \mu_{LT+1}}{\sigma_{LT+1}}\right) \cdot \sigma_{LT+1} \\
&= L\left(F^{-1}(CR)\right) \cdot \sigma \cdot \sqrt{LT+1} \\
&= L(1.769) \cdot 5.01 \cdot \sqrt{LT+1} \\
&= 0.0154 \cdot 5.01 \cdot \sqrt{LT+1}.
\end{aligned}$$

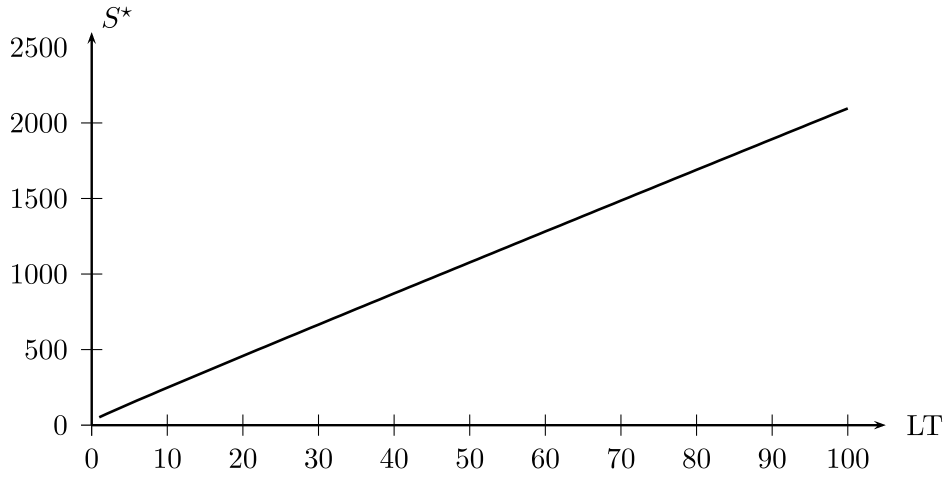
Hence we obtain

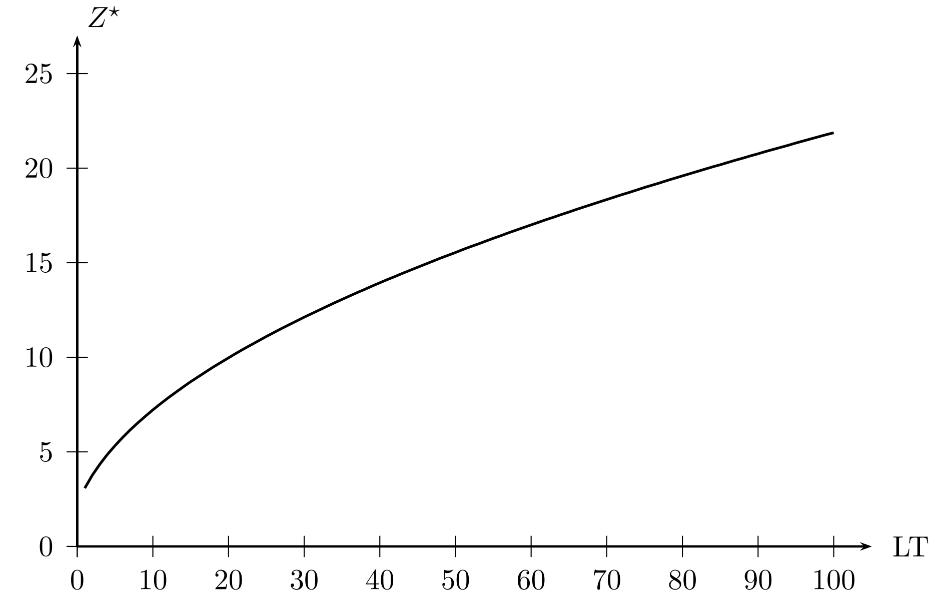
$$E\left[S - Y^{LT+1}\right]^+ = 1.769 \cdot \sqrt{LT+1} \cdot 5.01 + 0.0154 \cdot 5.01 \cdot \sqrt{LT+1} = 8.940 \cdot \sqrt{LT+1}.$$

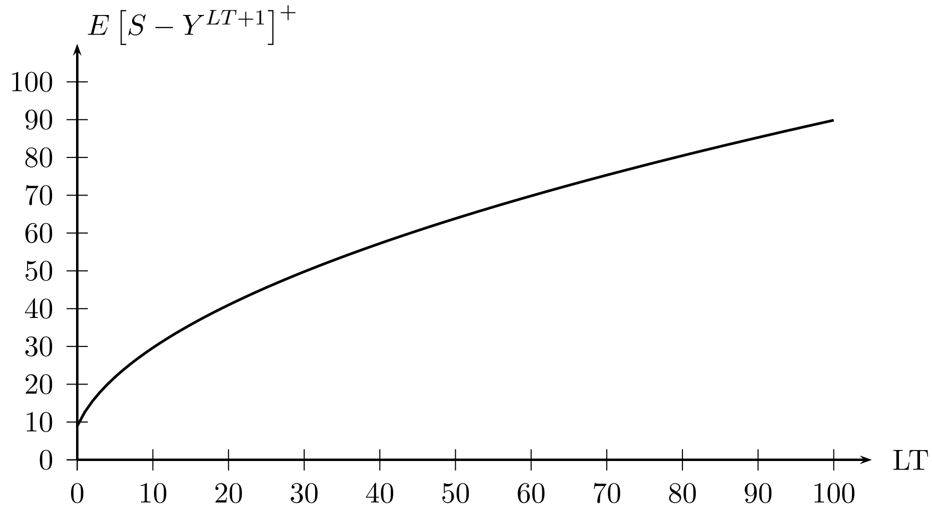
Optimal cost:

$$Z(S^\star) = 5.2 \cdot 0.0835 \cdot 5.01 \cdot \sqrt{LT+1}$$

v. See excel sheet.







vi. For $LT = 50$ we have already computed $Z(S^{\star}) = 15.54$. For $LT = 10$ we compute

$$Z(S^{\star}) = 5.2 \cdot 0.0835 \cdot 5.01 \cdot \sqrt{10+1} = 7.22$$

The cost advantage of $15.54 - 7.22 = 8.32$ per period is the maximum amount the wholesaler should be willing to pay.

vii. For $LT = 90$ we compute

$$Z(S^{\star}) = 5.2 \cdot 0.0835 \cdot 5.01 \cdot \sqrt{90+1} = 20.76$$

The wholesaler should request at least the cost disadvantage of $20.76 - 15.54 = 5.22$ as a compensation from the supplier.

Solution to Exercise 2:

i. See Excel spreadsheet. The forecast for the next year is $\hat{\mu} = 19.43$

ii. This value has already been computed in Exercise 1 i: $\sigma = 5.01$

iii. See Excel spreadsheet. The variance of the forecast error is 26.1133, the standard deviation is $\sqrt{26.1133} = 5.110$.

iv. In inventory control, we order to fill demand and future demand is estimated via forecasting. Safety stock is held to protect against the difference between our estimated demand value and the actual demand. This error is quantified by the forecast error. Consider an example, where demand is as follows:

Period	1	2	3	4	5	6	7
Demand	5	10	5	10	5	10	5
Forecast	5	10	5	10	5	10	5

We can perfectly forecast the demand, because it is seasonal but deterministic and there is no need to hold any safety stock. The forecast error is therefore always zero and so is the standard deviation of the forecast error. However, the standard deviation of demand is 2.67 and we would have to hold safety stock, if we use it in our inventory models.

v.
$$S^{\star} = F^{-1}(0.9615) = 19.43 \cdot (51) + 1.769 \cdot \sqrt{51} \cdot 5.110 \approx 1055$$